Announcements

HW10 due Friday (Solving hard problems using integer programming and algorithms with huge numbers used for crypto)

Upcoming plans:

Starting today: problems even much harder than NP-complete

Last homework: Hw11 will be due May 1st

Final (cumulative): Saturday May 9th at 9am.

Options for last few lectures (and other questions you may have): see survey.

Computability

- So far we have seen P, NP, EXP. There are many other interesting options, see video.

- Today: what is computable at all.

- What is computation? Church-Turing hypothesis

~ all models of computation
can compute the same things

Turing (1936) Turing machine extra simple language

Simple model of computation

simplified Java or Phyton

data types
- boolean
- characters (ASCII)
- integer (ignore size)
- array

expressions
- variables
- constans
- operations $+, <$
- length array

program
- return boolean

focus yes/no decision

statements
- assignments
- if ( ) then ...
- loops
- return

Note: a program is a string

input is also a string

Universal program (interpreter)

Universal ("program" "input") run "program" on given "input" & return answer

see 4120 next spring

Computational question:

running program on input what can happen

- returns 1 = yes
- returns 0 = no (or crashes)
- never stops

Halting problem: given program M input x

does M ever stop on x

Languages and decidability?

program M boolean output

language $L_M = \{$ input x s.t. M returns 1 on $x\}$
↘ accepts

define L is there a program M such that $L_M = L$

L recognizable is there is program $L_M = L$

decidable if M output 0 on $x \notin L$

Example $L = \{$all bipartite graph with perfect matchings$\}$

L is decidable – see matching algorithm we saw

Note: no longer care about time as long as algorithms stops in finite time

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Is the Hamiltonian Path problem decidable or recognizable?

A. Hamiltonian Path is decidable

B. Hamiltonian Path is recognizable, but not decidable

C. Hamiltonian Path is not even recognizable

n nodes
n! possible path
check each

recognizable: ✓
∃ alg returns yes
if graph has a
Hamiltonian path

decidable:
recognizable &
algorithm stops
& returns no if
no Hamiltonian path

The Halting problem

$L = \{$ program, input : program is "java" & program when run on input terminates $\}$

→ does not run forever

Halting problem

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Is the Halting problem decidable or recognizable?

given program M
Input x
does M terminate on x

A. Halting problem is decidable
B. Halting is recognizable but not decidable
C. Halting problem is not even recognizable

Run universal (M, x)
if M terminates return yes = 1
never returns "no", not a deciding algorithm

See proof that Halting not decidable later